WELCOME

3rd المؤتمر الدولي للذكاء الاصطناعي في تحقيق التنمية المستدامة

The Role of Artificial Intelligence in Sustainable Development

Bayesian Estimation of the Parameters Half-Normal Distribution Using Lindley's Approximation

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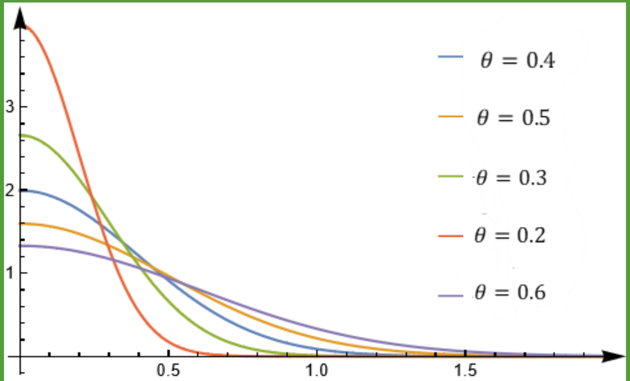
Half-Normal Distribution

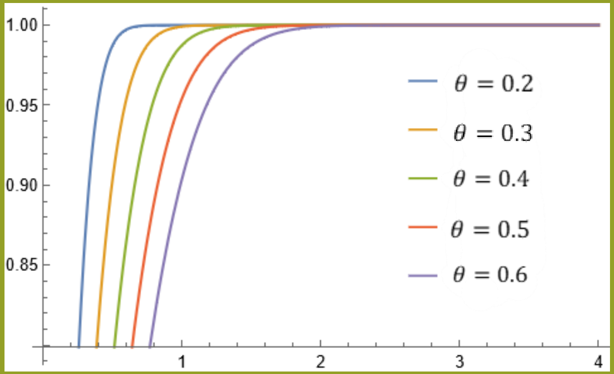


The graph of the sample with the PDF of the Half-Normal Distribution where as θ as be the variable with the following values $\{0.2, 0.3, 0.4, 0.5, 0.6\}$.

Probability Density Function (PDF) of the half-normal distribution is defined as follows:

$$f(x) = \sqrt{\frac{2}{\pi\theta^2}}\; e^{\frac{-x^2}{2\theta^2}} \quad , \qquad x \geq 0$$

Half-Normal Distribution



The graph of the sample with the CDF of the Half-Normal Distribution where as θ as be the variable with the following values $\{0.2, 0.3, 0.4, 0.5, 0.6\}$.

The cumulative distribution function (CDF):

$$F(x) = P(X \leq x)$$

$$= \int_0^x f(t)dt$$

$$= erf\left(\frac{x}{\theta\sqrt{2}}\right)$$

Lindley Approximation

His formula is given as follows

$$\mathbf{E}u(\theta)|S) \approx \hat{g} + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}(u_{ij} + 2u_i\rho_j)\sigma_{ij} + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}(L_{ijk}\sigma_{ij}\sigma_{kr}u_r)$$

Bayesian Estimation of the Parameters of the Gamma Distribution using Lindley's Approximation

The joint prior distribution:

We will take $\alpha \sim R(c_1)$ and $\beta \sim HN(c_2)$

$$P_1(\alpha) = \frac{\alpha e^{-\frac{\alpha^2}{2{c_1}^2}}}{{c_1}^2}$$

$$P_2(\beta) = \frac{2c_2 e^{-\frac{{c_2}^2\beta^2}{\pi}}}{\pi}$$

And α, β are independent

Then,

$$P(\alpha, \beta) = P_1 P_2$$

$$P(\alpha, \beta) = \frac{2\alpha c_2 e^{-\left(\frac{\alpha^2}{2{c_1}^2} + \frac{{c_2}^2\sigma^2}{\pi}\right)}}{{c_1}^2\pi}$$

we can have the log likelihood of a random sample of size n:

$$L = \sum_{j=1}^{n} \left(-\alpha\, log(\beta) - log\ (\Gamma(\alpha)) - log(x_j) + \alpha\, log(x_j) - \frac{x_j}{\beta}\right)$$

Which implies required derivatives of L:

$$L_{03} = \sum_{j=1}^{n} \left(-\frac{2\alpha}{\beta^3} + \frac{6x_j}{\beta^4}\right)$$

$$L_{30} = -n\, \psi(2, \alpha)$$

$$L_{12} = \frac{n}{\beta^2}$$

$$L_{21} = 0$$

we can have the logarithm of the joint prior distribution:

$$\rho = -\frac{\alpha^2}{2{c_1}^2} - \frac{{c_2}^2\beta^2}{\pi} + log(2) - 2\,log(c_1) + log(C_2) - log(\pi) + log(\alpha)$$

Derivative of ρ:

$$\rho_{10} = \frac{\partial\rho}{\partial\alpha} = \frac{\frac{e^{-\frac{\alpha^2}{2{c_1}^2}}}{{c_1}^2} - \frac{\alpha^2 e^{-\frac{\alpha^2}{2{c_1}^2}}}{{c_1}^4}}{\frac{\alpha e^{-\frac{\alpha^2}{2{c_1}^2}}}{{c_1}^2}}$$

$$\rho_{01} = \frac{\partial\rho}{\partial\beta} = -\frac{2\beta{c_2}^2}{\pi}$$

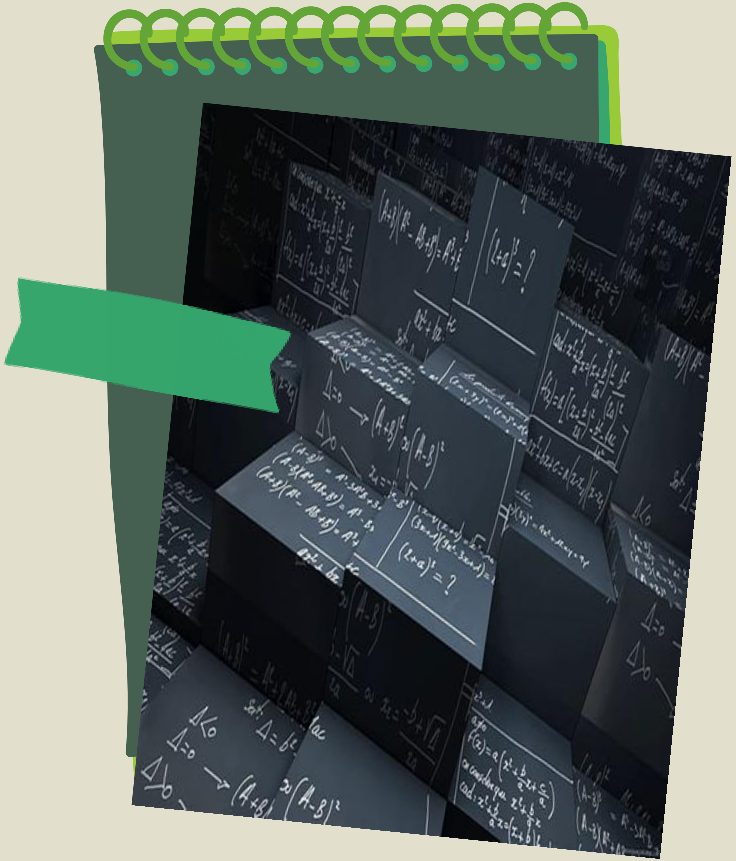
Fisher information matrix and its inverse $\widehat{FI}$ evaluated at $\hat{\alpha}$ and $\hat{\beta}$:

$$\sigma_{11} = -n\,\psi(1, \alpha)$$

$$\sigma_{22} = \sum_{j=1}^{n}\left(\frac{\alpha}{\beta^2} - \frac{2x_j}{\beta^3}\right)$$

$$\sigma_{12} = -\frac{n}{\beta}$$

$$\sigma_{21} = -\frac{n}{\beta}$$



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Example:

Applied Lindley approximation for mean of the gamma distribution $g(\alpha, \beta) = \alpha\, \beta$.

$$u_{12} = u_{21} = 1$$
$$u_{11} = u_{22} = 0$$
$$uu_1 = \beta$$
$$uu_2 = \alpha$$

The Bayes Lindley Approximated Estimate of $g(\alpha, \beta)$:

$$A_1 = (u_{10}\sigma_{11} + u_{01}\sigma_{12})$$

$$A_2 = (u_{01}\sigma_{22} + u_{10}\sigma_{21})$$

$$B_1 = (l_{30}\sigma_{11} + l_{21}\sigma_{12} + l_{21}\sigma_{21} + l_{12}\sigma_{22})$$

$$B_2 = (l_{21}\sigma_{11} + l_{12}\sigma_{12} + l_{12}\sigma_{21} + l_{03}\sigma_{22})$$

$$T_1 = (u_{20} + 2u_{10}\rho_{10})\sigma_{11}$$

$$T_2 = (u_{11} + 2u_{01}\rho_{10})\sigma_{21}$$

$$T_3 = (u_{11} + 2u_{10}\rho_{01})\sigma_{12}$$

$$T_4 = (u_{02} + 2u_{01}\rho_{01})\sigma_{22}$$

Therefore,

$$g_{BL} = \hat{g} + \frac{1}{2}(T_1 + T_2 + T_3 + T_4) + \frac{1}{2}(A_1B_1 + A_2B_2)$$

Conclusion

We have applied Bayesian techniques for estimating the parameters of the Half-Normal distribution the classical methods (maximum likelihood, method of moments) and the Bayesian method, where we used Lindley's approximation when we can't get Bayesian estimator in closed forms. We have further studied some characteristics of these estimators.

Thanks